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TECHNICAL NOTE

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A Calculation of the Theoretical Significance of Matched Bullets

ABSTRACT: The comparison and identification of bullets from the striations that appear on their surfaces, after they have been fired from a gun, have been practiced since the 1920s. Although the significance of the correspondences of these impression marks has been empirically justified, there is a conspicuous absence of any theoretical foundation for the likelihood. What is presented here is the derivation of the formulae for calculating the probability for the correspondence of the impression marks on a subject bullet to a random distribution of a similar number of impression marks on a suspect bullet of the same type. The approach to the calculation entails subdividing the impression marks into a series of individual lines having widths equal to the separation distance at which a misalignment of striations between the two bullets cannot be distinguished. This distance depends upon the resolution limit imposed by the microscope as well as by the visual acuity of the examiner. A calculation of the probabilities for finding pairs and triplets of consecutively matching lines on nonmatching bullets, by an examiner with normal perception using a microscope at 40× magnification, produces values that agree well with the empirical probabilities determined by Biasotti in the 1950s and when determined for larger consecutive sequences suggest that they are extremely unlikely to occur. The formulae can be used to determine the probabilities for the random occurrence of any sequence of striae and provide a straightforward way to quantitatively justify the significance of a specific match between any two bullets.

KEYWORDS: forensic science, firearms examination, impression evidence, statistical justification

The method of bullet identification is based on the identification of similar striations on the surface of a bullet when it is compared to those from one that has been test fired from the suspected gun. The technique involves using a comparison microscope where both bullets can be viewed simultaneously at a magnification of about 40×. The determination that a particular bullet was discharged from a specific gun is performed routinely by firearm examiners across the country and has been generally accepted as evidence in court for many years. Their justification for identification is drawn from the comparison of specific regions of similarity and some examiners have adopted the criterion of the presence of a specific number of consecutively matching striae as a way to reduce the level of subjectivity. When using this latter method, concluding it to be from the same gun is simply based upon whether such a correspondence has ever been reported for bullets discharged from different guns. The statistical likelihood that a particular correspondence of the striae will occur by chance has, however, never been properly assessed, leaving the method open to challenge in light of the Daubert ruling, which sets standards for the scientific validity of the evidence that can be presented in the courtroom.

Background

It has generally been accepted since the early 1900s that when a bullet travels through the barrel of a gun, a small proportion of the markings left on it provide a unique identifier, and can be used to establish that the bullet was fired from a particular firearm (1,2). The use of a comparison microscope to make such identifications has been standard since Goddard's work in the late 1920s (3,4), where the similarities between two bullets are compared directly. Over the years, the examiners have established for themselves what constitutes striae and what constitutes a match, and the conclusions they reach do not depend on theoretical criteria but rather depend on experience and training (5). The subjectivity of this process has been recognized for some time, and work to establish an objective criterion began in 1959 when Biasotti conducted a statistical study using twenty-four .38 caliber Smith and Wesson revolvers. He concluded that calculating the average percentage of matching striae could not provide a criterion for identification but that evaluating the number of consecutively corresponding striae could (6). He found that when lead bullets were fired from different weapons, sets of more than three consecutive matching striae were never found and when jacketed bullets were fired from different weapons, sets of no more than four consecutive striae were found. The precise criteria used can vary considerably, but currently when sets of striae with six or more components can be seen to match in approximately the same location on the projectile surface, the bullets are considered to have been derived from the same gun, even though the criteria derived by Biasotti are different from this (7). The idea that there is an absolute cut-off in terms of the number of consecutively corresponding striae that constitutes a match is of course unrealistic as Bunch points out (8) and although Brackett (9) made an early attempt to justify the concept his numerical models were incomplete. Nevertheless the technique of consecutive stria matching and the exponential decline in their occurrence as the length of the sequence increases has been repeatedly validated for different types of firearms (10,11). Studies of other types of abrasion damage have also shown the same type of behavior for nonmatching tool marks (12).

A 1997 review of the firearms identification literature by Nichols (13) summarized 34 articles and concluded that "all have had as a common concern the basis upon which identification in firearms and tool marks is achieved." Such a conclusion in light of the

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Daubert decision (14) emphasizes the necessity to determine an objective way to establish criteria for identification and provide justification of it in a court of law.

The Probabilities of Finding Matched Sequences of Striae

In making a direct comparison of a suspect bullet with an exemplar of known origin, the examiner evaluates the surfaces of the two bullets at 30–40× magnification in a comparison microscope and basically seeks to match the individual impression marks or striae that exhibit similar spacing and impression depth. Comparisons are typically performed by orienting to the leading edge of the land impression and examining the base of the bullet because this is where most of the carry over, or consistently reproducible impressions from the barrel, occurs. Land impressions are preferred because they are less affected by the step cutting broach or other swaging operations used to machine the barrel. The number of striae in a land impression can vary depending on the size of the bullet and the characteristics of the barrel. The Smith and Wesson barrel, for example, has a land impression width of 0.24 cm, and according to Biasotti's data has an average of 60 striae with total striae counts between 16 and 97.

The shortcoming of the consecutively matching striae method, that there is no theoretical justification for the cutoff criteria of six matching striae, can be appropriately addressed by looking at the possibility of finding correspondences in the random comparison of stria sequences. Such comparisons are relevant only if it can be proven that the firearms examiner has eliminated the striae associated with class characteristics and is comparing striae that are derived from the impressions due to random tool marks.

The development of a theoretical foundation for bullet comparisons, in accordance with matching particular sections of a land or groove impression, necessitates determining the number of times that a particular sequence of striae can occur on a random suspect bullet and dividing it into the total number of these patterns that actually exist on the subject bullet. This quantity is the probability that one of the patterns on the random bullet will match. Another way to look at this is that we are multiplying the number of possible sequences on the subject bullet with the probability for finding a particular one of the sequences on a random suspect bullet, to determine the likelihood that a match can occur by pure chance. Before these quantities can be determined, however, it is necessary to establish the maximum number of striae that can be accommodated on the land impression of the bullets and this first step requires a consideration of this in terms of what an examiner might be capable of distinguishing through the microscope.

The Assessment of the Maximum Number of Striae

When comparing two sets of striae on two different bullets at 40× magnification, the resolution through the eyepiece, r, is between 20 and 30 microns for most people. Under these conditions, we have a limited number of possibilities of coincidence because we cannot distinguish all the details of the striae and have to evaluate the coincidence in terms of what we can resolve at this resolution. This is equivalent to partitioning the impression into component lines 20–30 microns wide to comprise the striae we can distinguish. Assuming that we can resolve a 20-micron line for example, the narrowest striae we will be able to distinguish are 20-microns wide and we will also be able to distinguish only striae of increasing width in increments of 20 microns. Thus, when an examiner is trying to match particular sequences of striae, he is limited to a distinct set of line widths (i.e., 20, 40, 60,…), rather than a continuum. When the land impression has a width w, then the probability that we will find a particular line at a specific location within the land impression is $P_1 = r/w$. Thus, for a land impression that is 0.24 cm wide and examined at 40× magnification, by someone with normal perception, the chances of finding a line at a particular place on the circumference on the land is $P_1 = r/w = 20 \times 10^{-4}/0.24 = 0.0083$. That is to say the probability is 1 in 120, which means that there are 120 discernible line positions (Q) on the land, and any particular line that happens to be involved in a match to any other bullet will be found in one of these locations.

The Number of Sequences on a Random Suspect Bullet

The number of different ways that a sequence of n lines can be distributed over the Q locations is given by $\omega_{Qn} = Q!/(n!(Q-n)!)$, as the number of ways that N objects can be arranged in j subsets is $\omega = N!/(n_1!n_2!n_3!\cdots n_j!)$, where $N = \sum_{i=1}^{i=j} n_i$.

For the case of pairs of lines, for example, when there are 120 discernible line positions, $\omega_{Q_2} = \frac{Q!}{2!(Q-2)!} = \frac{120!}{2!118!} = 7140$, that is, to say there are over 7000 different ways to construct line pairs, and the probability of finding any specific one of them is

$$P_{Q_2} = \frac{1}{\omega_{Q_2}} = 0.00014$$

By way of an example, we might consider instead the simpler case of a very small section of the land impression where there are only five possible locations, as in Fig. 1. In this case, we can see by inspection that there are 10 ways in which pairs of lines can be distributed, which can be calculated from

$$\omega_{Q_2} = \frac{Q!}{2!(Q-2)!} = \frac{5!}{2!3!} = 10$$

Given that there are 10 different ways to arrange doublets, if we have a single doublet on another land that is also contained within 100 microns (five feature widths), then the probability that they will match when we put them together is

$$P_{Q_2} = \frac{1}{10} = 0.1$$

The figure also exemplifies the concept that two lines directly adjacent to another line are equivalent to a line of double thickness. Although it would seem that multiple line thicknesses might need to be treated separately this is not the case when the comparison is based upon meeting a specific level of resolution. This is because the probability for encountering a line of different thickness is actually the same as the probability for finding the requisite number of adjacent 20-micron lines that produce the same result. Another way to look at this is that random tool marks are by their very nature precluded from exhibiting any preference for either occurring together or separately. The immediate consequence of this to the firearms examiner is that the probabilities of consecutive striae sequences that he can distinguish are not all going to be equivalent. This Orwellian situation, where consecutive striae sequences were all considered to be equal but now some are apparently more equal than others, arises because the thicker striae are less likely to occur at random. Thus, the most common consecutive line sequences will be those involving the finest resolvable lines and these are the ones that form the basis for the empirical standards for the number of consecutive line sequences required for a match. This is possibly why some examiners are reluctant to use the

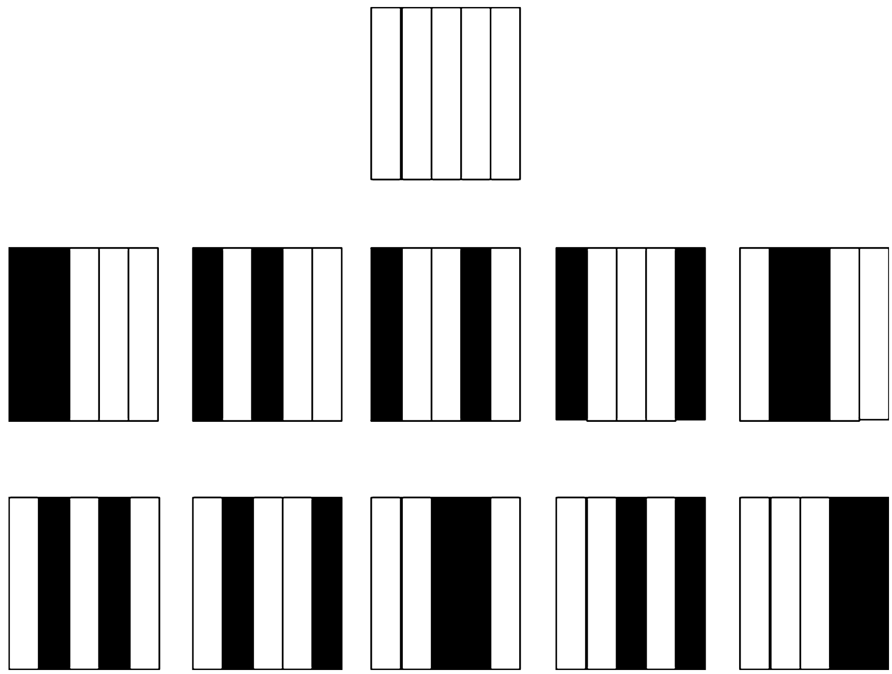

FIG. 1—*A schematic diagram showing the 10 different ways in which pairs of lines can be arranged within five possible locations. All 10 are obviously consecutive doublets.*

method because the criteria for a consecutive line match do not take into account the intricacies of the broad striae, even though the widths of the striae that are being matched are required to be the same. The consequence to the fact that lines of different thickness are equivalent to sets of smaller lines is that the probabilities of a particular correspondence do not actually depend upon the number of consecutive striae in the sequence; rather they are determined by the total number of matching lines that are contained within a region of correspondence. Thus, any region of coincidence on the two bullets, where the presence of striae appears in exactly the same way is simply reflecting a specific arrangement of 20-micron wide lines. For example if we count the number of striae on the land impression, or more precisely the total number of 20 micron units that produce them, and then determine the number of ways that these 20 micron units can be arranged as adjacent pairs on the surface of the bullet, we can then deduce the likelihood that we will find a specific adjacent pair at random.

In the case of the Smith and Wesson barrel, there are essentially 120 micron units and on average 61 distinguishable striae (6). The number of striae constitutes about half the number of possible locations where they can occur suggesting that most of the distinguishable striae have to be of the order of 20 microns wide. Therefore, it is not unreasonable to take the number of striae, which people have determined, rather than the number of 20 micron units that compose them, for the purposes of comparison, although in this paper, these calculations have been done over a broad range of line numbers (30–90). While it is obviously required for consecutive line matching, clearly any precisely matching pattern necessitates that the lines are not to be interrupted by any other lines and so will therefore be consecutive as well. To determine the probabilities for a pattern of lines to be consecutive, one must take into account the restrictions imposed by the other lines on the bullet and correct for them. A pair of lines with any additional lines between them does not, for example, constitute a consecutive doublet and so these sorts of doublets must not be included. We can calculate the total number of pairs among the lines, including the ones we do not want to count, by again applying the equation for the way N objects can be arranged in sets of n, i.e.,

$$\omega_n^N = \frac{N!}{n!(N-n)!}$$

For the case of three lines arranged in pairs (Fig. 2), the total number of pairs that can be formed between the lines is three because you can form pairs between lines 1 and 2, lines 2 and 3, and lines 1 and 3. The pair formed between lines 1 and 3 is not consecutive however, and we can deduce that by inspection. We can also consider the correction in general terms because when N lines are stacked next to each other there can only be $N - n + 1$ consecutive sequences of n lines.

We can therefore determine an expression for the number of consecutive line sequences that could exist among the lines by recognizing that the total number of sequences on a bullet that contains N lines is $\frac{N!}{n!(N-n)!}$ and that all of these are forbidden except for the $N - n + 1$ consecutive sequences that could actually be present, and so in the general case the number of forbidden sequences is given by

$$\omega_{\text{forbidden}} = \frac{N!}{n!(N-n)!} - (N - n + 1)$$

where n is the number of lines in the sequence.

The general expression for the number of consecutive lines that can exist is therefore

$$\omega_{\text{cl}}^n = \frac{Q!}{n!(Q-n)!} - \frac{N!}{n!(N-n)!} + (N - n + 1)$$

and the nine consecutive doublets among the 10 possible pairs can be determined by inspection in Fig. 3, where the lines have simply been renumbered for purposes of identification.

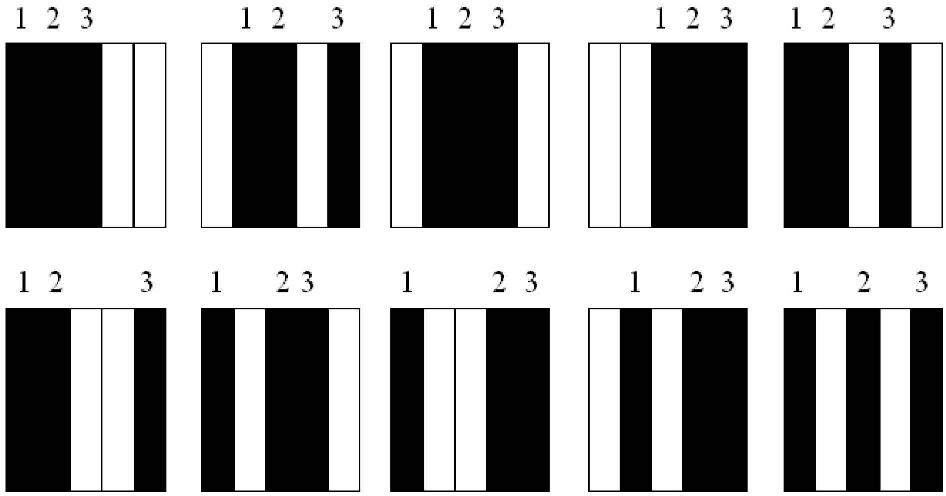

FIG. 2—*A schematic showing the possibilities for three lines distributed among five locations. 1&2 and 2&3 constitute doublets but 1&3 do not, regardless of the distribution of the lines. The number of forbidden combinations in all these cases is 1.*

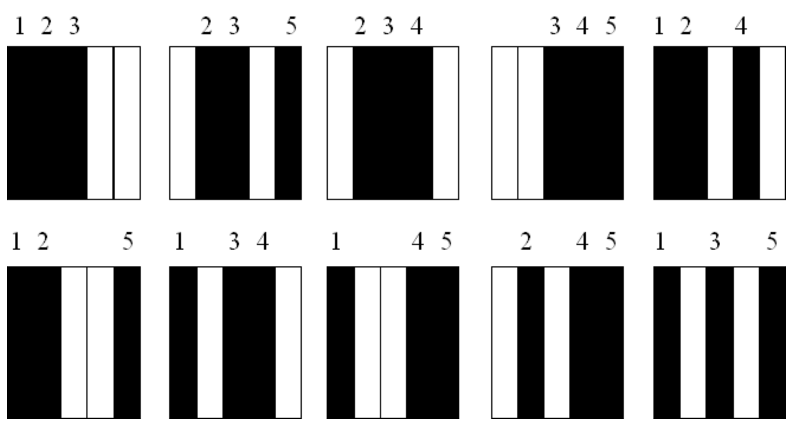

FIG. 3—*A schematic showing the 10 possible combinations of three lines distributed among the five labeled locations. For the case of three lines on a bullet with five positions, there are 10 possibilities but only nine consecutive pairs. The nine distinguishable pairs, some of which appear more than once, only occur in locations 1&2, 2&3, 3&4, 4&5, 2&4, 2&5, 1&3, 1&4, and 3&5; they cannot occur in location 1&5.*

For the more realistic case of the doublet, when there are 60 lines on the bullet, the potential number of consecutive line possibilities is

$$\omega_{\rm cl}^2 = \frac{120!}{2!(118)!} - \frac{60!}{2!(58)!} + (60 - 2 + 1) = 5429$$

The probability of finding a specific consecutive doublet when there are 60 lines on the bullet with 100 possible positions is the inverse of the total number of different sequences that are possible multiplied by the fraction of the sequences that are consecutive and is given by

$$P_{\rm cl}^2 = \frac{\omega_{cl}^2}{\omega_{Q_2}} P_{Q_n} = \frac{\omega_{\rm cl}^2}{(\omega_{Q_2})^2} = \frac{5429}{(7140)^2} = 0.000106$$

Another way to look at this is that about 76% of the doublets could be consecutive. In summary, therefore, the number of sequences of n consecutive lines that could exist on a bullet with N lines distributed among Q possible locations is

$$\omega_{\rm cl}^n = \frac{Q!}{n!(Q-n)!} - \frac{N!}{n!(N-n)!} + (N - n + 1)$$

The total number of different sequences is $\omega_{Qn} = \frac{Q!}{n!(Q-n)!}$

The probability that any particular sequence of n lines that we choose beforehand on the subject bullet and then find on a random bullet is

$$P_{Qn} = \frac{n!(Q-n)!}{Q!}$$

and the probability that the sequence is consecutive is

$$\begin{aligned} P_{cl}^n &= \frac{\omega_{\rm cl}^n}{\omega_{Q_n}} P_{Q_n} \\ &= \left(\frac{n!(Q-n)!}{Q!}\right)^2 \left(\frac{Q!}{n!(Q-n)!} - \frac{N!}{n!(N-n)!} + (N - n + 1)\right) \end{aligned}$$

or

$$P_{\rm cl}^n = P_{Q_n} - (P_{Q_n})^2 \left(\frac{N!}{n!(N-n)!} - (N - n + 1)\right)$$

The values for P_{Q_n} and $P_{\rm cl}^n$ are shown in Tables 1 and 2 for the Smith and Wesson barrel at a resolution of 20 microns for different numbers of lines on a bullet. These are the probabilities that n lines, located at random on a bullet, will match to a specific one of the individual set of n lines (P_{Q_n}) or a specific one from the set of consecutive lines ($P_{\rm cl}^n$) on the subject bullet. It is

TABLE 1—*The calculated probabilities for finding random sequences of line pairs on the land of a bullet for different values of the sequence* n *and the width of the land in millimeters.*

	$P_{Qn} = \frac{n!(Q-n)!}{Q!}$				
Land Width	20 mm	21 mm	22 mm	23 mm	24 mm
Doublet	0.0002	0.00018	0.00016	0.00015	0.00014
Triplet	6.18×10^{-06}	5.33×10^{-06}	4.63×10^{-06}	4.05×10^{-06}	3.56×10^{-06}
Quadruplet	2.55×10^{-07}	2.09×10^{-07}	1.73×10^{-07}	1.45×10^{-07}	1.22×10^{-07}
Quintuplet	1.32×10^{-08}	1.04×10^{-08}	8.17×10^{-09}	6.52×10^{-09}	5.25×10^{-09}
Sextuplet	8.38×10^{-10}	6.21×10^{-10}	4.67×10^{-10}	3.55×10^{-10}	2.74×10^{-10}
Septuplet	6.24×10^{-11}	4.39×10^{-11}	3.14×10^{-11}	2.28×10^{-11}	1.68×10^{-11}
Octuplet	5.37×10^{-12}	3.59×10^{-12}	2.44×10^{-12}	1.69×10^{-12}	1.19×10^{-12}

TABLE 2—*The calculated probabilities for finding random consecutive sequences of lines on a 24mm wide land for different values of the sequence and the total number of lines on the impression.*

	$P^n_{\mathrm{cl}} = P_{Q_n} - (P_{Q_n})^2\left(\frac{N!}{n!(N-n)!} - (N-n+1)\right)$				
Line Number	30	40	50	60	70
Doublet	0.000132	0.000126	0.000117	0.000107	9.41×10^{-5}
Triplet	3.50×10^{-6}	3.44×10^{-6}	3.31×10^{-6}	3.13×10^{-6}	2.87×10^{-6}
Quadruplet	1.21×10^{-7}	1.20×10^{-7}	1.18×10^{-7}	1.15×10^{-7}	1.08×10^{-7}
Quintuplet	5.24×10^{-9}	5.23×10^{-9}	5.19×10^{-9}	5.10×10^{-9}	4.91×10^{-9}
Sextuplet	2.73×10^{-10}	2.73×10^{-10}	2.73×10^{-10}	2.70×10^{-10}	2.64×10^{-10}
Septuplet	1.68×10^{-11}	1.68×10^{-11}	1.68×10^{-11}	1.67×10^{-11}	1.65×10^{-11}
Octuplet	1.19×10^{-12}	1.19×10^{-12}	1.19×10^{-12}	1.19×10^{-12}	1.18×10^{-12}

notable that the difference between the probabilities for consecutive lines and other patterns are not very different and that these numbers are relatively constant over a fairly large range of line numbers. The probabilities decrease as the number of lines approach the number of possible line locations, which will happen if the resolution is poor, but for the case of a typical land impression, where the microscope magnification is 40× and the number of lines is between 30 and 70% of the number of resolvable locations, they are relatively similar.

The Number of Sequences on the Subject Bullet

We can use the same expressions we used to determine the number of line sequences that could exist among the lines on the random bullet for the lines that are known to exist on the subject bullet; that is, the total number of sequences of n on a bullet that contains N lines is $\frac{N!}{n!(N-n)!}$ and the number of consecutive sequences is $N - n + 1$.

The Total Probabilities

When the subject bullet is compared to the random bullet, we will have to take into account all the simultaneous comparisons that are being made, and for this we look to the product of the probability for finding the sequence on the random bullet with the number of sequences that are actually present on the subject bullet. The subject bullet is actually the reason why consecutive line sequences have been more popular than regular line sequences because there are many fewer consecutive sequences to match. The number of individual sequences typically being in the thousands for doublets is generally given by $\omega_n^N = N_2!/(n!(N_2-n)!)$, although the number of consecutive line sequences is less than the number of lines $\omega_{cl}^N = (N_2 - n + 1)$. These probabilities for the two types of coincidence are shown in Tables 3 and 4. To compare these values directly to Biasotti's data, the number of land impressions compared to each other in the analysis must be taken into account, and as Smith and Wesson revolvers have five land impressions,

TABLE 3—*The calculated probabilities for finding random line sequences that match in the comparison of single land impressions on bullets fired from Smith and Wesson revolvers.*

	$P_{Q_n}\omega^n_{N_2}$				
	30	40	50	60	70
Doublet	0.061	0.109	0.172	0.248	0.338
Triplet	0.014	0.035	0.069	0.122	0.195
Quadruplet	0.0033	0.0111	0.0280	0.0594	0.1116
Quintuplet	0.00075	0.00345	0.01112	0.0287	0.0635
Sextuplet	0.00016	0.00105	0.00435	0.0137	0.0359
Septuplet	3.42×10^{-5}	0.00031	0.00168	0.0065	0.0202
Octuplet	6.96×10^{-4}	9.15×10^{-5}	0.00064	0.0030	0.0112

presumably 25 comparisons were evaluated. The probabilities for finding the line correspondences from 25 comparisons are shown in Table 5 and the probabilities for finding the various consecutive sequences are shown in Table 6.

According to Biasotti, for lead bullets and jacketed bullets, respectively, the probabilities were 0.2 and 0.46 for a doublet and 0.01 and 0.1 for a triplet. For between 30 and 70 lines on a bullet, these probabilities range from 0.1 to 0.16 for a doublet and from 0.003 to 0.005 for a triplet at 20-microns resolution, and at 30 microns they range from 0.14 to 0.24 for a doublet and from 0.007 to 0.01 for a triplet.

These probabilities are plotted in Fig. 4 in the manner of Biasotti and in Fig. 5 over a logarithmic range of probability. The probabilities for the sequences when consecutiveness is not required are shown in Fig. 6, and these probabilities are considerably more dependent on the number of lines on the bullet, which is probably why finding large numbers of corresponding striae has been deemed a poor criterion for matching.

Conclusions

The close similarity between the results of these calculations and the experimental observations by Biasotti, of the chance occurrence

TABLE 4—*The calculated probabilities for finding random consecutive line sequences that match in the comparison of single land impressions on bullets fired from Smith and Wesson revolvers.*

	$P^n_{\mathrm{cl}}\omega^{cl}_{N_2}$				
	30	40	50	60	70
Doublet	0.0038	0.0049	0.0057	0.0063	0.0065
Triplet	9.82×10^{-5}	0.00013	0.00015	0.00018	0.00019
Quadruplet	3.27×10^{-6}	4.45×10^{-6}	5.56×10^{-6}	6.53×10^{-6}	7.25×10^{-6}
Quintuplet	1.36×10^{-7}	1.88×10^{-7}	2.39×10^{-7}	2.85×10^{-7}	3.24×10^{-7}
Sextuplet	6.84×10^{-9}	9.57×10^{-9}	1.23×10^{-8}	1.49×10^{-8}	1.72×10^{-8}
Septuplet	4.03×10^{-10}	5.71×10^{-10}	7.38×10^{-10}	9.02×10^{-10}	1.05×10^{-9}
Octuplet	2.73×10^{-11}	3.93×10^{-11}	5.11×10^{-11}	6.29×10^{-11}	7.41×10^{-11}

TABLE 5—*The calculated probabilities for finding random line sequences that match in the comprehensive evaluation of two bullets fired from Smith and Wesson revolvers.*

	$25P_{Q_n}\omega^n_{N_2}$				
	30	40	50	60	70
Doublet	1.52	2.73	4.29	6.19	8.45
Triplet	0.36	0.88	1.75	3.05	4.87
Quadruplet	0.08	0.28	0.70	1.48	2.79
Quintuplet	0.02	0.08	0.28	0.72	1.58
Sextuplet	0.004	0.03	0.11	0.34	0.89
Septuplet	0.0009	0.007	0.04	0.16	0.50
Octuplet	0.0002	0.002	0.02	0.07	0.28

TABLE 6—*The calculated probabilities for finding random consecutive line sequences that match in the comprehensive evaluation of two bullets fired from Smith and Wesson revolvers.*

	$25P^n_{\mathrm{cl}}\omega^{cl}_{N_2}$				
	30	40	50	60	70
Doublet	0.096	0.122	0.143	0.157	0.162
Triplet	0.0025	0.0033	0.0039	0.0045	0.0049
Quadruplet	8.19×10^{-5}	0.00011	0.00014	0.00016	0.00018
Quintuplet	3.41×10^{-6}	4.71×10^{-6}	5.97×10^{-6}	7.14×10^{-6}	8.11×10^{-6}
Sextuplet	1.71×10^{-7}	2.39×10^{-7}	3.07×10^{-7}	3.71×10^{-7}	4.29×10^{-7}
Septuplet	1.01×10^{-8}	1.43×10^{-8}	1.85×10^{-8}	2.25×10^{-8}	2.64×10^{-8}
Octuplet	6.84×10^{-9}	9.82×10^{-9}	1.28×10^{-9}	1.57×10^{-9}	1.85×10^{-9}

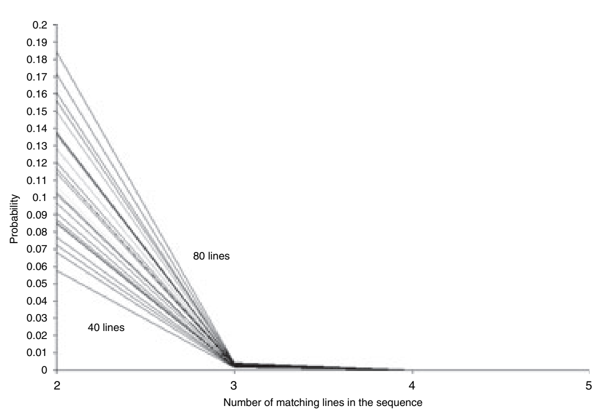

FIG. 4—*Probabilities of random consecutive line sequences from a Smith and Wesson bullet comparison plotted in the manner of Biasotti.*

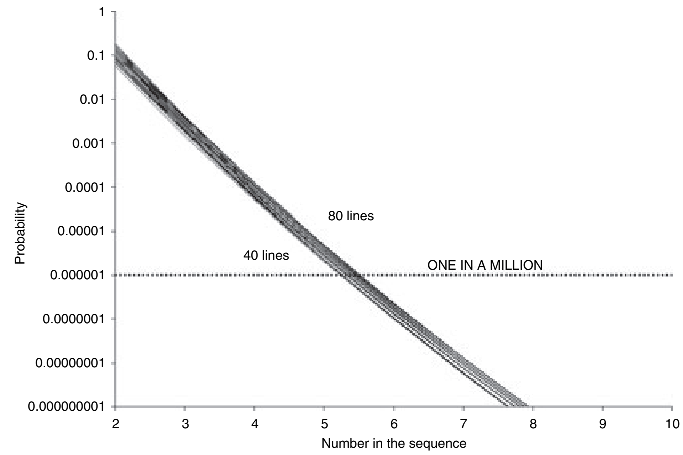

FIG. 5—*A logarithmic plot of the probabilities of random consecutive line sequences as they vary with the total number of lines on a bullet from a Smith and Wesson revolver.*

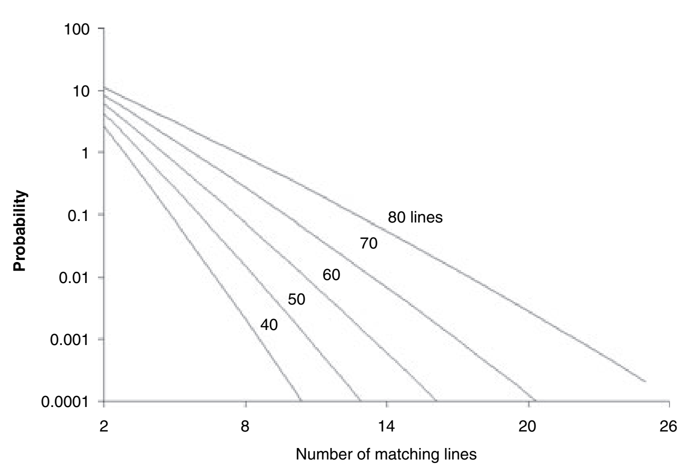

FIG. 6—*A logarithmic plot of the probabilities of random line sequences as they vary with the total number of lines on a bullet from a Smith and Wesson revolver.*

of small numbers of consecutive stria correspondence, appear to be in reasonable agreement. It is also apparent that consecutive stria matching, although a useful way to standardize the different approaches used by firearms examiners, is actually a less rigorous case of congruent pattern matching. This is exemplified by the observation made earlier, that the probabilities for consecutive line matches are not all equivalent and depend upon the width of the striae. This is because a line twice as thick as the finest line that can be resolved is actually a consecutive doublet and one three times as thick is a consecutive triplet and so on. For example, comparing two bullets each with 50 lines on the land impression,

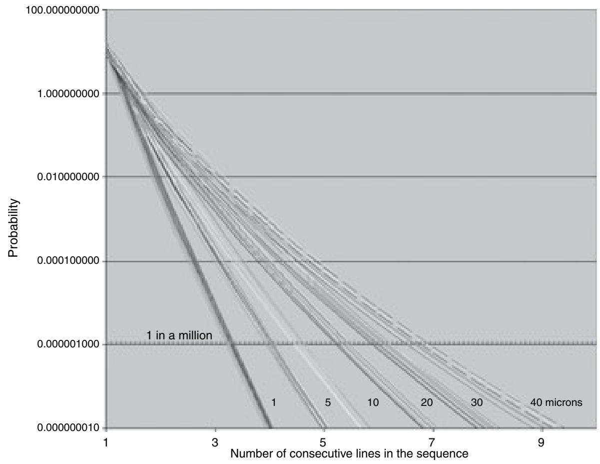

FIG. 7—*A logarithmic plot of the probabilities of random consecutive line sequences as they vary with the resolution in microns on a bullet from a Smith and Wesson revolver with 30 lines on it.*

a consecutive line sequence of two, composed of two single 20-micron wide lines, resolvable at 40× magnification with a space in-between would have a probability of 0.143 according to Table 6. Were one of the lines to be twice as thick, however, the sequence now corresponds to a sequence of three consecutive lines and not two, which from the same table has a probability of 0.0039, and if both lines were twice as thick this would correspond to a sequence of four with a probability of 0.00014. The approach taken here, to consider a line thicker than the minimum that can be resolved as a sequence of consecutive lines is actually a calculation of the absolute probability of a one-dimensional pattern match and is therefore quite general. Alternatively, one could consider them as single lines, and then determine the probabilities that a line should be of a particular width and multiply the two probabilities together, but as long as the lines are random the two results should be the same. What can be drawn from this conclusion is that to maximize the level of certainty associated with a correspondence, rather than simply counting a sequence of lines, the examiner should determine the extent of the matching correspondence in terms of the contributing lines and their width. The patterns in Fig. 2, for example, would be consecutive line sequences of equivalent validity and yet traditionally only one of them would be classified as a consecutive triplet and three of them would not be considered to be significant. For comparison purposes, when more than one region of correspondence is found, examiners might also wish to consider the probabilities in total and present the product of the probabilities for finding each of the correspondences separately.

Another individual task the examiners may want to do is determine the resolution at which they can comfortably distinguish a line width in their microscope. Although we have essentially adopted the resolution criterion for the distinction of line pairs by most people, which is 20–30 microns, with some of the modern microscopes at 40× magnification it is possible for some of the examiners to resolve features down to 5 microns and thereby increase their confidence in a match even further. The resolution being lower is not a cause for concern until the number of lines that can be resolved start to approach the number of lines that are on the bullet. In Fig. 7, for example, the probabilities for the different consecutive line sequences are plotted as a function of resolution for the case where there are 30 lines on a bullet. At a resolution of 15 microns, 160 distinct lines can be distinguished on the land impression of a Smith and Wesson revolver but at 60-microns resolution only 40 are distinguishable. Although the probabilities for the doublet change very little over this range, those of the longer sequences change by several orders of magnitude.

In light of these observations, it is hardly surprising that the debate about whether one should adopt consecutive line matching continues. In adopting what has been a significant body of work, to empirically justify the observations of Biasotti in 1950, essentially involves abandoning the details of the striae that are clearly significant to a subjective determination, but are hard to justify. Hopefully this method for calculating the probabilities, based upon the number of features that can be distinguished as a line, will enable all firearms examiners to quantify their subjective observations into discreet probabilities, regardless of how they choose to validate their observations empirically.

Summary

The matching of recurring patterns of striae on the surface of bullets has been widely used to determine whether they were fired from the same gun. Although these matches are not usually associated with particular likelihood ratios, this is certainly possible to do with a few single measurements such as the width of the land, the magnification used, and the number and width of striae on the bullets being compared. Such quantitation should provide a reliable minimum estimate of the likelihood of finding such a match by chance.

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Appendix: Definition of Terms

- r The smallest linear dimension that can be resolved using the microscope.
- w The width of the land impression on the bullet.
- P_1 The probability of a line being found at a specific location on the land.
- Q The number of locations resolved within the land.
- n The number of lines in the sequence.
- N The number of lines on the suspect bullet.
- N_1 The number of lines on the subject bullet of known origin.
- ω_{Qn} The number of ways an n line sequence can be distributed over the Q locations.
- ω_{cl}^{n} The potential number of n consecutive line sequences on the land.
- ω_{cl}^{N} The number of different consecutive n line sequences among N lines.
- ω_{n}^{N} The number of different n line sequences among N lines.
- $\omega_{\mathrm{forbidden}}$ The number of different nonconsecutive n line sequences among N lines.
- P_{Q_n} The probability of finding a particular n line sequence on the land.
- P_{cl}^{n} The probability of finding a particular consecutive n line sequence on the land.